Indian Institute of Science, Bangalore

UE 204: Endsemester Exam

Date: 26/4/19.
Duration: 2.00 p.m.–5.00 p.m.
Maximum Marks: 100

Note: Throughout the paper, you can neglect the transverse shear contribution to the energy. If no geometrical or material properties are given, assume A, J, I to be the area, polar moment of inertia and area moment of inertia, and E and G to be the Young modulus and shear modulus. You may directly use the formulae at the back.

1. A disc of inner radius a, outer radius b, and unit width along the z-direction is subjected to equal and opposite moments M on the outer and inner boundaries $r = b$ and $r = a$ by the application of suitable tangential tractions which are independent of θ (see Fig. 1). This problem is identical to the one in the second test except that instead of the inner boundary being fixed, there is a now a moment applied at the inner boundary. The radial displacement is given by (25)

$$u_r = c_1 \cos\theta + c_2 \sin\theta + \frac{c_3}{r},$$

where c_1, c_2 and c_3 are constants to be determined. Assuming $u_z = 0$, the *average* u_θ over the domain to be zero, and treating the problem as two-dimensional (i.e., $u_r = u_r(r,\theta)$ and $u_\theta = u_\theta(r,\theta)$), find the displacement components (u_r, u_θ) in terms of $(r, \theta, \lambda, \mu, M, a, b)$. Make reasonable assumptions and state them clearly. The governing equations are

$$0 = (\lambda+\mu)\frac{\partial(\operatorname{tr}\boldsymbol{\epsilon})}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(ru_r)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_r}{\partial\theta^2} + -\frac{2}{r^2}\frac{\partial u_\theta}{\partial\theta}\right],$$

$$0 = (\lambda+\mu)\frac{1}{r}\frac{\partial(\operatorname{tr}\boldsymbol{\epsilon})}{\partial\theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(ru_\theta)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial\theta^2} + +\frac{2}{r^2}\frac{\partial u_r}{\partial\theta}\right].$$

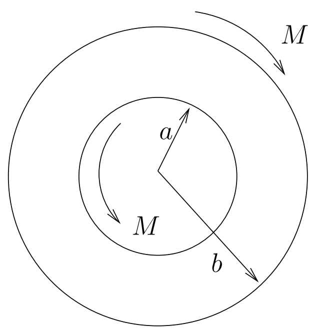

Figure 1: Hollow disk subjected to equal and opposite moments M on the outer and inner boundaries.

The strain-displacement and constitutive relations are

$$\epsilon_{rr} = \frac{\partial u_r}{\partial r}, \qquad \epsilon_{r\theta} = \frac{1}{2}\left[\frac{1}{r}\frac{\partial u_r}{\partial \theta} + r\frac{\partial}{\partial r}\left(\frac{u_\theta}{r}\right)\right],$$

$$\epsilon_{\theta\theta} = \frac{1}{r}\left(\frac{\partial u_\theta}{\partial \theta} + u_r\right), \qquad \operatorname{tr}\boldsymbol{\epsilon} = \epsilon_{rr} + \epsilon_{\theta\theta},$$

$$\boldsymbol{\tau} = \lambda(\operatorname{tr}\boldsymbol{\epsilon})\boldsymbol{I} + 2\mu\boldsymbol{\epsilon}.$$

2. In the setup shown in Fig. 2, a semicircular beam of radius R is fixed at one end, while the other end is joined to a horizontal beam AB of length L (30)

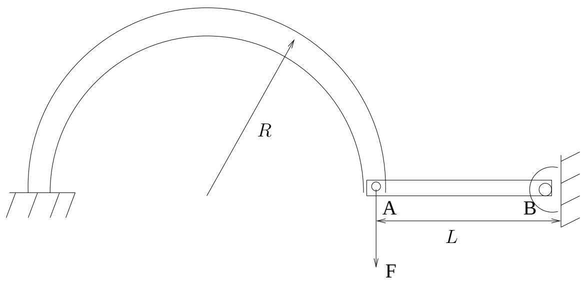

Figure 2: Problem 2.

by means of a pin joint. The member AB has pin joints at both ends. The semicircular and straight beams have identical cross sections (i.e., same A, J etc.). Find the stress in the member AB due to the application of the force F.

3. A belt is passed over one end of a circular cylinder of radius R that is fixed to a rigid wall at the other end as shown in Fig. 3. One end of this belt is fixed, while a force F is applied at the other end. The angle over which the belt makes contact with the cylinder is π. The coefficient of friction between the belt and the cylinder is μ, and the governing equation for the tension in the belt is (30)

$$\frac{dT}{d\theta} = \mu T.$$

Using the maximum shear stress criterion for the cylinder, i.e., the maximum shear stress in the cylinder is less than or equal to $Y/2$, where Y is given, find the allowable load F using a factor of safety of unity. Assume that the usual formulae for bending, torsion etc. for the circular cylinder hold, and that the width of the belt is negligible.

4. A double cantilever beam of length L is initially straight. The right rigid wall is moved upward by a distance v_0 as shown in Fig. 4. Find the reactions at the right end using (15)

   (a) the method of superposition. The displacement and slope at the end of a cantilever beam of length L due to load P are $PL^3/(3EI)$ and $PL^2/(2EI)$, while those due to a moment M are $ML^2/(2EI)$ and $ML/(EI)$.

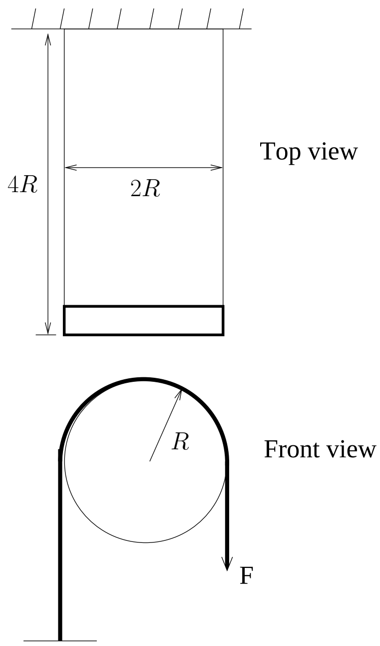

Figure 3: Problem 3.

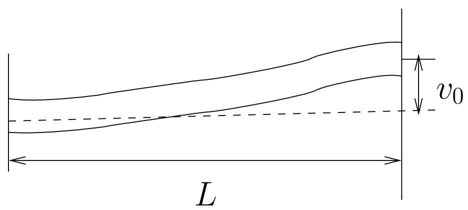

Figure 4: Problem 4.

(b) the differential equation

$$EI\frac{d^4v}{dx^4} = q(x).$$

Some relevant formulae

$$\int_0^\pi \sin^2\theta\,d\theta = \int_0^\pi \cos^2\theta\,d\theta = \frac{\pi}{2},$$
$$\int_0^\pi \sin\theta\,d\theta = 2,$$
$$\int_0^\pi \sin\theta\cos\theta\,d\theta = \int_0^\pi \cos\theta\,d\theta = 0.$$