ME261: ODE Assignment 1
Aug–Dec 2018
Assignment 1 Due Oct. 16, 2018

Answer the questions below. If appropriate, please box your final expression/ result.

Question 1: Solving IVPs [*20 marks*]

Solve the following initial value problems. Assume a suitable domain for t. Wherever an initial condition is not specified, leave your final answer in terms of the constant of integration

$$t\frac{dx}{dt} + (1+t)x = 3 \tag{1}$$

$$\cos(t)\frac{dx}{dt} + \sin(t)x + \sin^2(t) = 1 \tag{2}$$

$$(3x^2 + 3x\cos(x) + 1)\frac{dx}{dt} + \frac{1}{1+t^2}(2tx + x) = 0 \tag{3}$$

$$t\frac{dx}{dt} + (t+1)x + tx^5 = 0 \qquad x(1) = 1 \tag{4}$$

$$t^2\frac{dx}{dt} = x^2 + tx \qquad x(1) = 1 \tag{5}$$

Question 2: Phase diagrams and bifurcations [*10 marks*]

Consider the following autonomous differential equation:

$$\frac{dx}{dt} = \lambda x - x^3 \qquad x(0) = x_0 \tag{6}$$

Based on our discussion in class, answer the following questions:

(a) Find the fixed points x^* for this system.

(b) Plot the phase diagram/portrait assuming $\lambda = 1$. If $x_0 = 1.5$, what do you expect will happen to the solution $x(t)$ as $t \to \infty$? What about $x_0 = 0.75$?

(c) For $\lambda > 0$, perform linear stability analysis for each of the fixed points x^* you found in part (a). Deduce which fixed points are stable and which are not. How will your answer change when $\lambda < 0$?

(d) Using all of the information above, plot the fixed points x^* as a function of λ. What is the bifurcation point? How is it different from the saddle-node case we discussed in class?