ME261: Assignment 3

Due: 28/10/16

In all the problems below, *verify* that the solution that you get satisfies the governing differential equation or equations.

1. By directly using the formulae given in the notes, find the general solution to the system of equations

$$\boldsymbol{v}' + \boldsymbol{A}\boldsymbol{v} = \boldsymbol{f},$$

where the prime denotes a derivative with respect to the independent variable t for the following two cases:

$$\boldsymbol{A} = \begin{bmatrix} -1 & -2 \\ -2 & -1 \end{bmatrix}, \quad \boldsymbol{f} = \begin{bmatrix} 2e^{4t} \\ e^{4t} \end{bmatrix},$$
$$\boldsymbol{A} = \begin{bmatrix} -2 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}, \quad \boldsymbol{f} = \begin{bmatrix} e^{t} \\ 0 \\ e^{-t} \end{bmatrix}.$$

2. Find the integrating factor and solve the following linear first order differential equations (*do not* directly use the formulae in the notes)

$$y' + \frac{1+x}{x} y = \frac{1}{x},$$
$$y' + (\tan x) y = \cos x.$$

3. Find all solutions to the following separable nonlinear ordinary differential equations

$$(3y^3 + 3y\cos y + 1)y' + \frac{(2x+1)y}{1+x^2} = 0,$$
$$(\sin x)(\sin y) + (\cos y)y' = 0.$$

4. Find the integrating factor and solve the following nonlinear differential equations

$$(3xy + 2y^2 + y)dx + (x^2 + 2xy + x + 2y)dy = 0,$$
$$(12xy + 6y^3)dx + (9x^2 + 10xy^2)dy = 0.$$

5. Find the general solution of the following second order equations

$$y'' + (y')^2 = x^2 + 1,$$
$$y'' + (y')^2 = (y^2 + 1)y',$$
$$y'' - 8y' + 16y = 23\cos x - 7\sin x,$$
$$y'' + y' = -8\cos 2x + 6\sin 2x,$$
$$y'' - 2y' + 2y = -6\cos 3x + 6\sin 3x,$$

$$y'' + 6y' + 13y = 18\cos x + 6\sin x,$$
$$x^2y'' + xy' + 4y = 2\tan(\log|x|),$$
$$y'' + \frac{2(x^3-1)}{x}y' + x^4y = 0,$$
$$4x^2y'' - 4xy' + (4x^2+3)y = x^{7/2}, \quad y_1 = \sqrt{x}\sin x,\ y_2 = \sqrt{x}\cos x,$$
$$(\sin x)y'' + (2\sin x - \cos x)y' + (\sin x - \cos x)y = e^{-x}, \quad y_1 = e^{-x},\ y_2 = e^{-x}\cos x,$$
$$xy'' - y' - 4x^3y = 8x^5, \quad y_1 = e^{x^2},\ y_2 = e^{-x^2}.$$