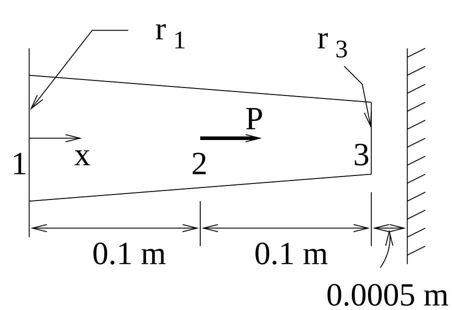
ME257: Assignment 2

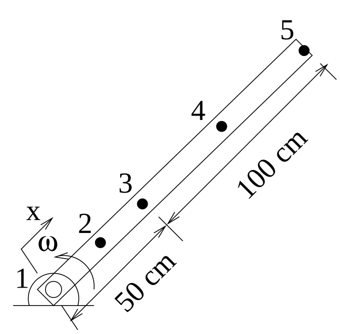
Due: 15/2/2016

1. The tapered bar shown in Fig. (a) has a Young's modulus $E = 60$ GPa, and is loaded by an axial load $P = 60000$ N. Assume the displacements to be a function of x alone, and that $u_y = u_z = 0$, i.e., $\boldsymbol{u} = u(x)\boldsymbol{e}_x$. The radius at any point along the bar varies linearly, and at points 1 and 3 is given by $r_1 = 15/\sqrt{\pi}$ mm, $r_3 = 5/\sqrt{\pi}$ mm, respectively. The free end of the bar is at a distance of 0.5 mm from a rigid wall. Formulate the element stiffness matrix using natural cordinates. Using a two-element (3 node) model and assuming that the wall is absent, find whether the free end makes contact with the wall under the given loading conditions. If it does, modify the boundary conditions appropriately and find the displacements at the nodes, the reactions at the walls, and the stresses in the elements.

2. A rod of length 150 cm rotates about a point with a constant angular velocity as shown in Fig. (b). The density, Young's modulus, cross sectional area, and angular velocity are given by $\rho = 7500$ kg/m^3, $E = 60$ GPa, $A = 4{\times}10^{-4}$ m^2, and $\omega = 20^c$/s. Assume $\boldsymbol{u} = u(x)\boldsymbol{e}_x$, and the loading to be only due to the centrifugal body force (given by $b = \rho x \omega^2$). Formulate the element stiffness matrix, and the consistent load vector for a *quadratic* (3-node) element. Then using the two-element (5-node) model shown in the figure, determine the displacements and the stress distribution in the rod using two quadratic elements. Plot the displacement and stress distributions (both exact and approximate) as a function of x. What do you observe about the stress distribution at node 3?



(a)



(b)